EEE 1131: Electrical Circuits

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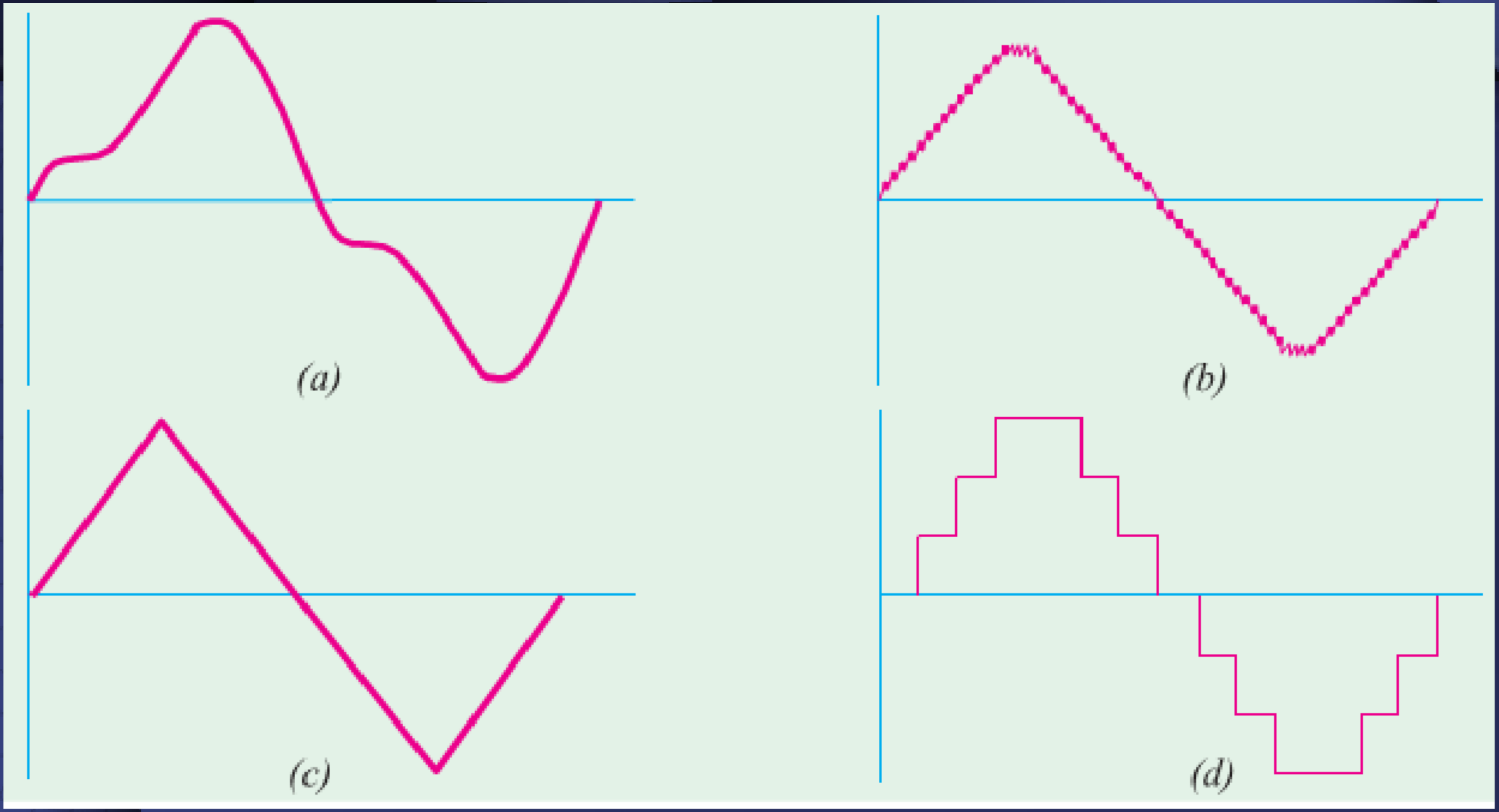
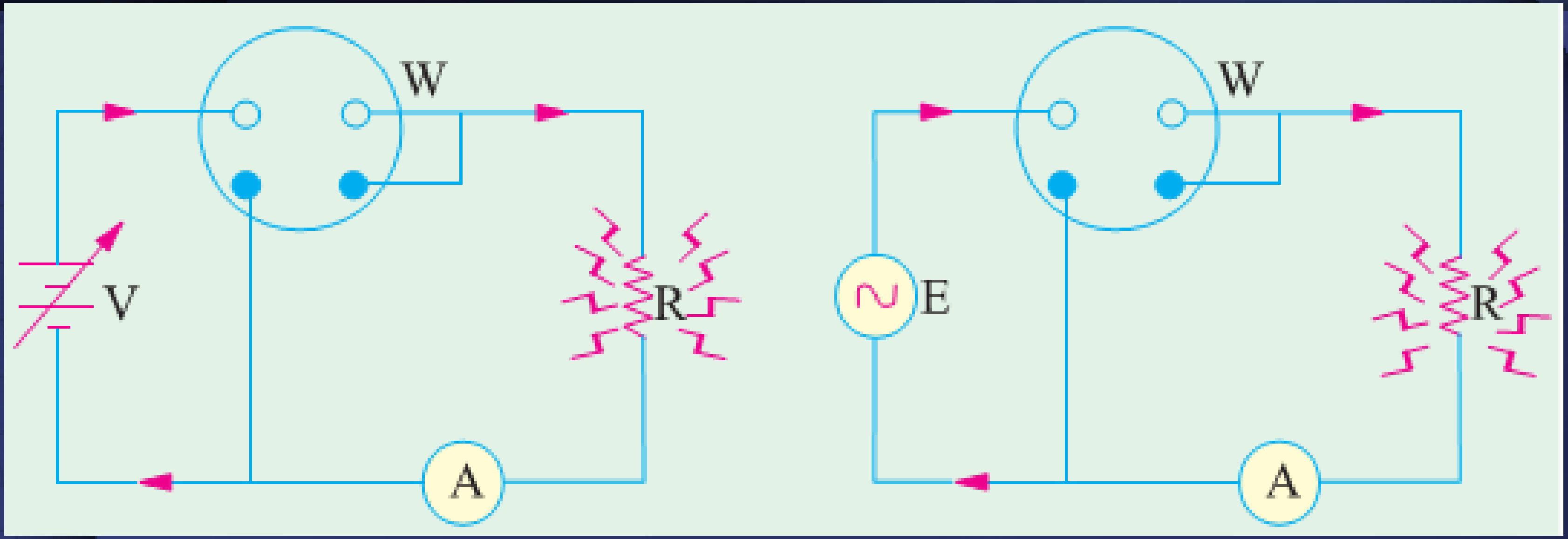
Waveform or Waveshape



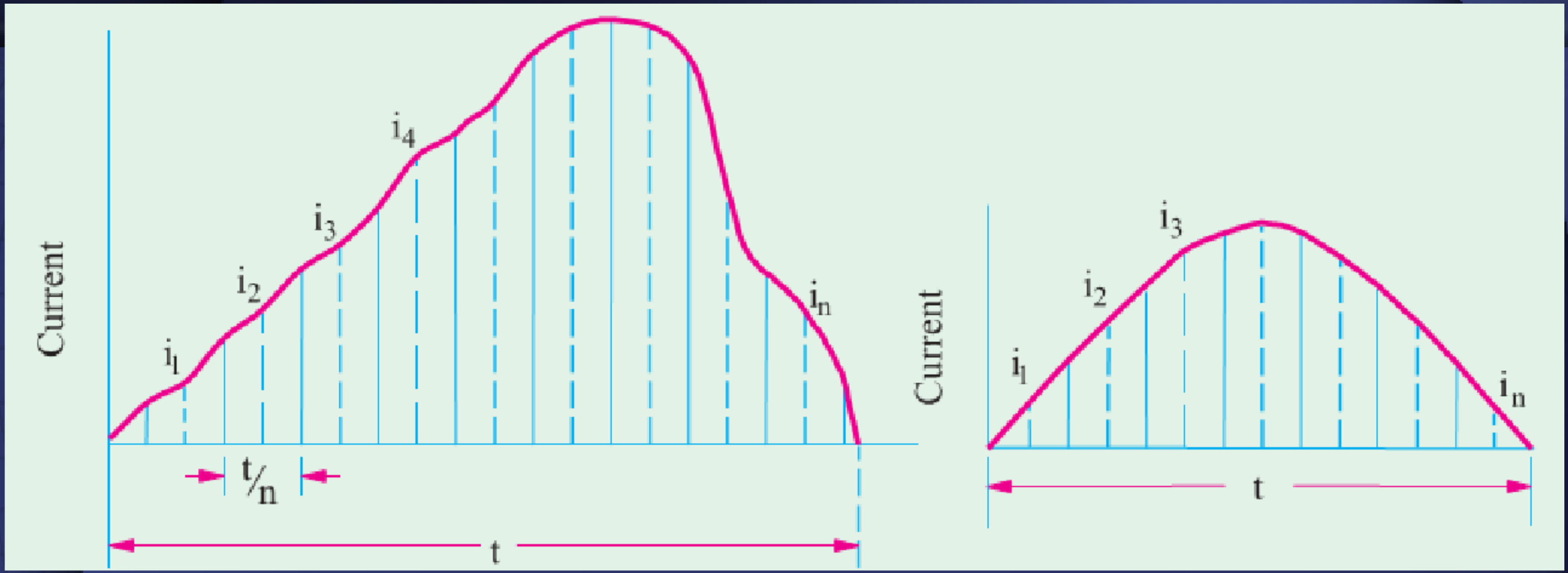
In general, however, ***an alternating current or voltage is one the circuit direction of which reverses at regularly recurring intervals.***

Root Mean Square Value



The r.m.s. value of an alternating current is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.

Mid Ordinate Method



$$I = \sqrt{\left(\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n}\right)}$$

$$V = \sqrt{\left(\frac{v_1^2 + v_2^2 + \ldots + v_n^2}{n}\right)}$$

Analytical Method

$$i = I_m \sin \omega t = I_m \sin \theta.$$

The standard form of a sinusoidal alternating current

$$\sqrt{\left(\int_0^{2\pi} \frac{i^2 \, d\theta}{2\pi}\right)}$$

$$I = \sqrt{\left(\int_0^{2\pi} \frac{i^2 \, d\theta}{2\pi}\right)} = \sqrt{\left(\frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2 \theta \, d\theta\right)}$$

the r.m.s. value of the alternating current

$$I = \sqrt{\left(\frac{I_m^2}{4\pi} \int_0^{2\pi} (1 - \cos 2\theta) \, d\theta\right)} = \sqrt{\left(\frac{I_m^2}{4\pi} \left| \theta - \frac{\sin 2\theta}{2} \right|_0^{2\pi}\right)}$$

$$= \sqrt{\frac{I_m^2}{4} \quad 2} \qquad \sqrt{\frac{I_m^2}{2}} \qquad \therefore \; \mathrm{I} = \frac{I_m}{\sqrt{2}} = 0.707 \, I_m$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

Analytical Method

r.m.s. value of current = 0.707 × max. value of current

The r.m.s. value of an alternating current is of considerable importance in practice, because the ammeters and voltmeters record the r.m.s. value of alternating current and voltage respectively

In electrical engineering work, unless indicated otherwise, the values of the given current and voltage are always the r.m.s. values.

It should be noted that the average heating effect produced during one cycle is

$$= I^2 R = (I_m/\sqrt{2})^2 R = \frac{1}{2} I_m^2 R$$

Average Value

The average value I of an alternating current is expressed by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.

In the case of a symmetrical alternating current (i.e. one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero

Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only.

But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.

Average Value

$$I_{av} = \frac{i_1 + i_2 + \ldots + i_n}{n}$$

Mid-ordinate Method

$$I_{av} = \int_0^{\pi} \frac{i d\theta}{(\pi - 0)} = \frac{I_m}{\pi} \int_0^{\pi} \sin\theta \, d\theta$$

Analytical Method

$$\frac{I_m}{\pi} \left| -\cos\theta \right|_0^{\pi} = \frac{I_m}{\pi} \left| +1 - (-1) \right| = \frac{2I_m}{\pi} = \frac{I_m}{\pi/2} = \frac{\text{twice the maximum current}}{\pi}$$

$$I_{av} = I_m / \tfrac{1}{2}\pi = 0.637\, I_m$$

average value of current = 0.637 × maximum value

Form Factor

$$K_f = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{0.707\ I_m}{0.637\ I_m} = 1.1.\ \text{(for sinusoidal alternating currents only)}$$

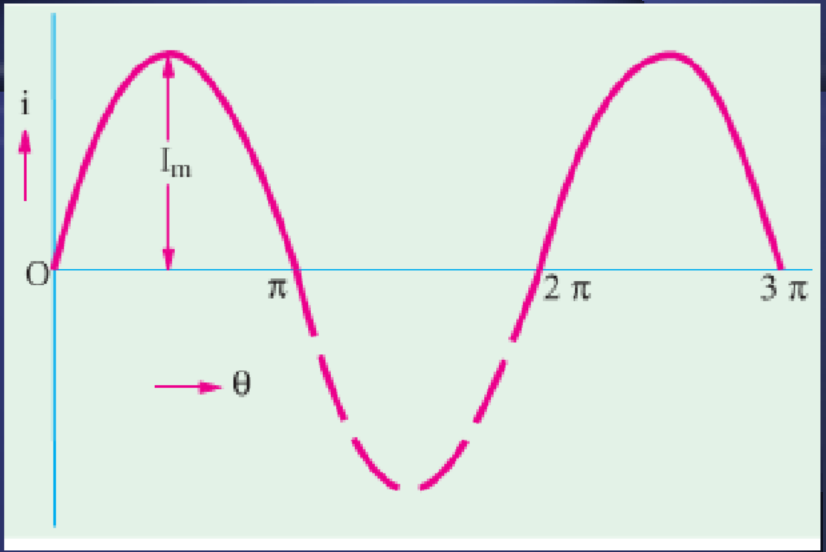
Crest or Peak or Amplitude Factor

$$K_a = \frac{\text{maximum value}}{\text{r.m.s. value}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414\ \text{(for sinusoidal a.c. only)}$$

R.M.S. Value of H.W. Rectified Alternating Current

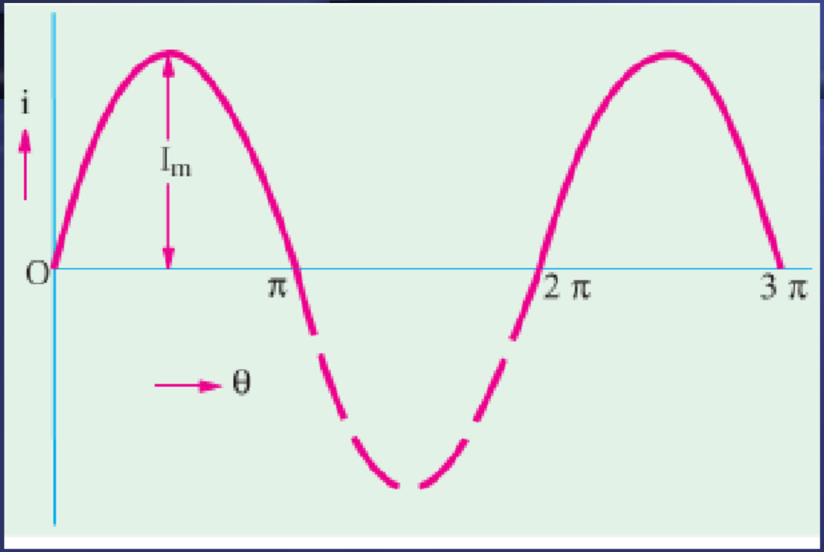
$$I = \sqrt{\left(\int_0^{\pi} \frac{i^2 d\theta}{2\pi}\right)} = \sqrt{\left(\frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta\right)}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{\pi}(1-\cos 2\theta)\,d\theta}$$



$$= \sqrt{\left(\frac{I_m^2}{4\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{\pi}\right)} = \sqrt{\left(\frac{I_m^2}{4\pi}\times\pi\right)} = \sqrt{\left(\frac{I_m^2}{4}\right)} \quad \therefore I = \frac{I_m}{2} = \mathbf{0.5I_m}$$

Average Value of H.W. Rectified Alternating Current



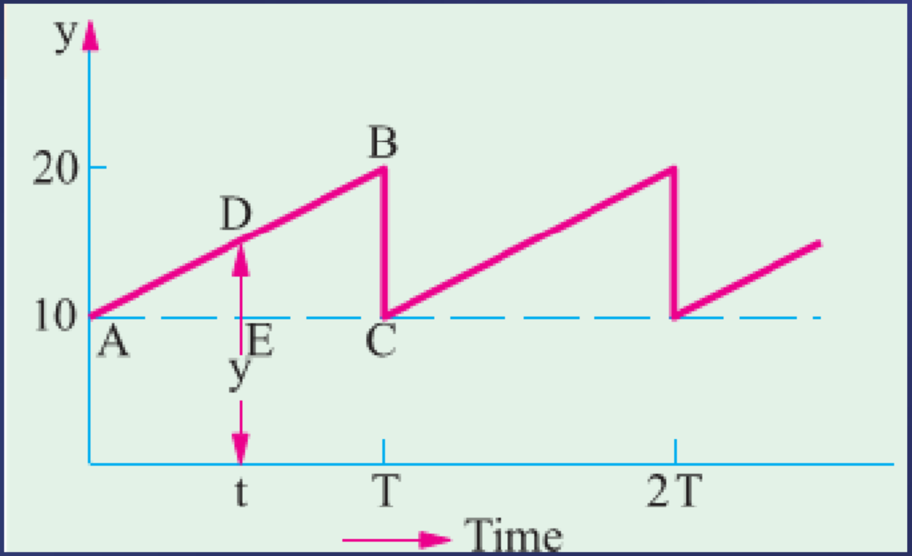
$$I_{av} = \int_0^{\pi} \frac{i d\theta}{2\pi} = \frac{I_m}{2\pi} \int_0^{\pi} \sin\theta \, d\theta$$

$$= \frac{I_m}{2\pi} \left| -\cos\theta \right|_0^{\pi} = \frac{I_m}{2\pi} \times 2 = \frac{I_m}{\pi}$$

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{I_m/2}{I_m/\pi} = \frac{\pi}{2} = 1.57$$

Numerical Problem

Example 11.13. *What is the significance of the r.m.s and average values of a wave ? Determine the r.m.s. and average value of the waveform shown in Fig. 11.25*



$$(y - 10)/t = 10/T$$

$$y = 10 + (10/T)t$$

Numerical Problem

$$Y_{av} = \frac{1}{T}\int_0^T y\,dt = \frac{1}{T}\int_0^T \left(10 + \frac{10}{T}t\right) dt$$

$$= \frac{1}{T}\int_0^T \left[10.dt + \frac{10}{T}.\,t.\,dt\right] = \frac{1}{T}\left|10t + \frac{5t^2}{T}\right|_0^T = \mathbf{15}$$

$$\text{Mean square value} = \frac{1}{T}\int_0^T y^2 dt = \int_0^T \left(10 + \frac{10}{T}t\right)^2 dt$$

$$\frac{1}{T}\int_0^T \left(100 + \frac{100}{T^2}t^2 + \frac{200}{T}t\right)dt = \frac{1}{T}\left|100t + \frac{100t^3}{3T^2} + \frac{100t^2}{T}\right|_0^T = \frac{700}{3}$$

$$\text{RMS value} = 10\sqrt{7/3} = \mathbf{15.2}$$

THANK YOU